Fluid Neutral Momentum Transport Reference Problem

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1 Summary

Type of problem: kinetic or fluid neutral transport

Physics or algorithm stressed: thermal force term (spatial resolution) in momentum transport equation and treatment of collisions (charge exchange)

Dimensionality & temporal variation: 1-D in space; steady state.

Type of solution: semi-analytic

Relevance: Divertor plasmas can be cool and dense enough for atoms to be in or near the fluid regime.

2 Detailed Description

2.1 Equations Solved

The fluid equations in Ref. [1], derived under the assumption of a constant charge exchange cross section, contain terms representing the thermal force and diffusion thermoeffect. To isolate these effects, we ignore the terms corresponding to viscosity, ionization, and recombination, although these processes may be significant in a physical situation. The resulting fluid neutral continuity and momentum balance equations in a slab geometry are

$$\frac{d}{dx}\left(Nv\right) = 0 \tag{1}$$

$$\frac{d}{dx}\left(mNv^2 + NT\right) = \alpha_T N \frac{dT}{dx} - m\nu_{\rm cx} Nv, \tag{2}$$

where N is the neutral density, v its flow velocity, T is its temperature (assumed equal to the ion temperature T_i), and m is the ion and neutral

mass. The thermal force coefficient, $\alpha_T = 0.24$, and the charge exchange frequency, $\nu_{\rm cx} = 2.93\sigma_{\rm cx} n_i (T/m)^{1/2}$, where n_i is the ion density.

2.2 Specific Details

2.2.1 Geometry

The specific geometry used for solving Eq. (2) is an $L = 1$ m long box.

2.2.2 Plasma Parameters

A linear ion temperature profile is assumed, ramping up from $T_i(0) = 1$ eV to $T_i(L) = 10$ eV. The ion density is computed from a constant pressure assumption,

$$n_i(x) = (n_{\rm min} T_{\rm max})/T_i(x), \tag{3}$$

where we take $n_{\rm min} = 8.5 \times 10^{18}$ m^{-3}.

As noted above, the charge exchange cross section is assumed to be a constant. The particular value we use, $\sigma_{\rm cx} = 4 \times 10^{-19}$ m^2, has been chosen to roughly correspond to the actual (*not* constant in general) charge exchange cross section [2] used in neutral transport simulations.

Note that the dimensionless parameter $l_{\rm mfp}/L$ needs to be small enough for the fluid approximation to be valid. Here, $l_{\rm mfp} = (T/m)^{1/2}/\nu_{\rm cx}$ is the charge exchange mean free path ($\propto 1/n_i$), A practical criterion is $\max(l_{\rm mfp}/L) \leq 0.1$ ("max" refers to the maximum value over the problem space).

2.2.3 Boundary Conditions

The $x = 0$ surface is a neutral entrance and $x = L$ is an exit. The simulation would be run in 1-D or equivalent.

2.2.4 Sources and Sinks

The neutral source corresponds to a gas puff at $x = 0$ of $S = 10^{19}$ H atoms per second. Note, however, that the data provided here are normalized so that the actual magnitude of the flux is irrelevant. The atoms have a Maxwellian energy distribution with a temperature of 1 eV [i.e., matching the value of $T_i(0)$].

2.3 Description of Solution Method

With a neutral source on one end of the slab ($x = 0$) and an exit at the other ($x = L$), Eq. (1) implies that the flux $N(x)v(x) \equiv j(x) = S/A$ (A is the effective cross sectional area of the slab) is constant. The exit represents a non-fluid boundary condition, meaning that we will only be able to find approximate solutions to Eq. (2).

First, consider a standard fluid approach in which we take $N(L) = 0$ and $mv^2 \ll T$. Equation (2) can then be integrated directly to yield

$$\frac{N_1(x)}{N_1(0)} = \left[\frac{T(x)}{T(0)}\right]^{\alpha_T - 1} \left\{1 - \frac{\int_0^x dx'\, \nu_{\rm cx}(x')\,[T(x')]^{-\alpha_T}}{\int_0^L dx'\, \nu_{\rm cx}(x')\,[T(x')]^{-\alpha_T}}\right\}. \tag{4}$$

We can plug this solution back into Eq. (2) and solve for the v on the right-hand side (again neglecting v^2 on the left hand side):

$$v(x) = \frac{T(x)^{1-\alpha_T}}{m} \left\{\int_x^L dx'\, \nu_{\rm cx}(x')\,[T(x')]^{-\alpha_T}\right\}^{-1}. \tag{5}$$

In particular, this can be evaluated at $x = 0$, allowing us to relate the neutral density there to the source strength

$$N_1(0) = S/[v(0)A], \tag{6}$$

permitting an absolute comparison of this profile with one obtained from some other code. However, as we will discuss further below, comparing absolute densities will, in general, be less insightful than comparing normalized density profiles.

A second approach is to extend the fluid model by writing the exiting flux as $j(L) = \gamma N(L)[T(L)/m]^{1/2}$, where $\gamma \sim 1$ ($\gamma < 1$) is an undetermined numerical factor. We expect the results to be insensitive to the precise value of γ; this point will be discussed in detail in Sec. 3. Equation (2) can then be written in terms of the relative density $\eta \equiv N_2(x)/N_2(L)$,

$$\frac{d\eta}{dx} = -\left\{\frac{\frac{(1-\alpha_T)}{T(L)}\frac{dT(x)}{dx}\eta + \frac{\gamma\nu_{\rm cx}}{[T(L)/m]^{1/2}}}{\frac{T(x)}{T(L)} - \frac{\gamma^2}{\eta^2}}\right\}, \tag{7}$$

and numerically integrated from $x = L$ to $x = 0$ using specified plasma temperature profiles.

Alternatively, the parameter γ can be eliminated by defining $\eta' \equiv N_2/[\gamma N_2(L)]$,

$$\frac{d\eta'}{dx} = -\left\{ \frac{\frac{(1-\alpha_T)}{T(L)}\frac{dT(x)}{dx}\eta' + \frac{\nu_{\rm cx}}{[T(L)/m]^{1/2}}}{\frac{T(x)}{T(L)} - \frac{1}{\eta'^2}} \right\}, \tag{8}$$

In terms of the source strength, the absolute density is then

$$N(x) = \frac{S}{A[T(L)/m]^{1/2}}\eta'(x). \tag{9}$$

Note also that

$$N(L) = \frac{S}{A\gamma[T(L)/m]^{1/2}}. \tag{10}$$

Once the integrations in Eqs. (4) and (8) have been carried out, we could use Eqs. (6) and (9), respectively, to determine the corresponding absolute density profiles for a particular S. However, the better approach when comparing a simulation code with these results is to first focus on the normalized profiles. E.g., in the second solution γ controls the rate at which particles flow out the exit. For a given S, a smaller γ corresponds to a lower exit speed and, by flux conservation, a higher overall density. Consequently, the absolute $N(x)$ profile (for fixed S) is sensitive to the value of γ, which we do not know a priori. But, by normalizing the density profile, say, to a particular $N(0)$, all of the variation with γ is confined to the region near $x = L$. This effectively corresponds to choosing a value of S that yields that particular density at that normalization point. If the simulation result and $N_2/N(0)$ agree satisfactorily for $x < L$, then the value of γ can be varied to improve the agreement near $x = L$.

Even after the value of γ has been optimized, the value of S corresponding to this $N(0)$ [via Eq. (9)] will differ from that used in a kinetic simulation code. The particle velocity distribution function associated with Eq. (9) at $x = L$ is only an approximation to the one that obtained by a fully kinetic treatment of the exit boundary condition. Correspondingly, the average (flow) velocity at $x = L$ will not be the same in the two cases. Hence, to get the two densities to match at $x = L$ (or anywhere else), we will need different particle fluxes, i.e., values of S.

3 Model Results

The text file associated with this document contains the input plasma profiles as well as results corresponding to the approximate fluid solutions, Eqs. (4) and (7). Figure 1 is a plot of these same data. Note that the mesh is uniformly spaced, except for "guard" cells near the ends. The uniform spacing allows the spatial resolution to be easily characterized while the "guard" cells provide us with solution values near $x = 0$ and $x = L$.

The integral in Eq. (4) is computed using the trapezoidal rule. Equation (8) is integrated numerically using the `LSODE` [3] routine from $x = L$ to $x = 0$ using $\eta'(L) = 1/\gamma$ as an initial condition; for this reason, $1/\gamma$ is a more convenient parameter to work with than γ. Here, we take $1/\gamma = 1.01$.

Note that the two approaches to a solution rely on different normalizations. To simplify comparison with simulation results, as noted in Sec. 2.3, we re-normalize the second solution, from Eq. (8), so that it has the same value as $N_1(x)/N_1(0)$ at the first value of x; we refer to this as $N_2(x)/N(0)$ below and in Fig. 1.

Without a more accurate solution, we can only provide plausible estimates of the errors in these solutions. First, since we suspect that $N_2(x)/N(0)$ is more accurate than $N_1(x)/N_1(0)$, we use the difference between the two solutions as the error bars for the latter.

Second, we set the error bars of $N_2(x)/N(0)$ using its variation with $1/\gamma$. As was pointed out in Sec. 2.3, we expect $1/\gamma \sim 1$ (and $1/\gamma > 1$); i.e., the exit speed is close to the sound speed at $x = L$. We find that for $1/\gamma > 2$ the resulting $N_2(x)/N(0)$ depends noticeably on $1/\gamma$. Since this means that the boundary condition is being "felt" well upstream, we suspect that the "correct" value of $1/\gamma$ is < 2. We then use the resulting range of $1/\gamma$ to set the error bars on $N_2(x)/N(0)$. In particular, the error is equated to $N_2(x; 1/\gamma = 1.01)/N(0) - N_2(x; 1/\gamma = 2)/N(0)$.

`x`	= distance from source in meters.
`N1(x)/N1(0)`	= Result of first analytic model, normalized to 1 at $x = 0$.
`N1/N1(0)err`	= Estimated error in result of first analytic model.
`N2(x)/N(0)`	= Result of second analytic model normalized to same value at $x = 5 \times 10^{-4}$ m as the first analytic model.
`N2/N(0)err`	= Estimated error in result of second analytic model.
`Ti(x)(eV)`	= ion temperature in eV.
`ni(x)(m^-3)`	= ion density in m^{-3}.

References

[1] P. Helander et al., Phys. Plasmas **1** 3174 (1994).

[2] R. K. Janev and J. J. Smith, At. Plasma-Mater. Interaction Data Fus. **4** 1 (1993).

[3] A. C. Hindmarsh, "ODEPACK, A Systematized Collection of ODE Solvers", in **Scientific Computing**, R. S. Stepleman et al. (Eds.), (North-Holland, Amsterdam, 1983), p. 55.

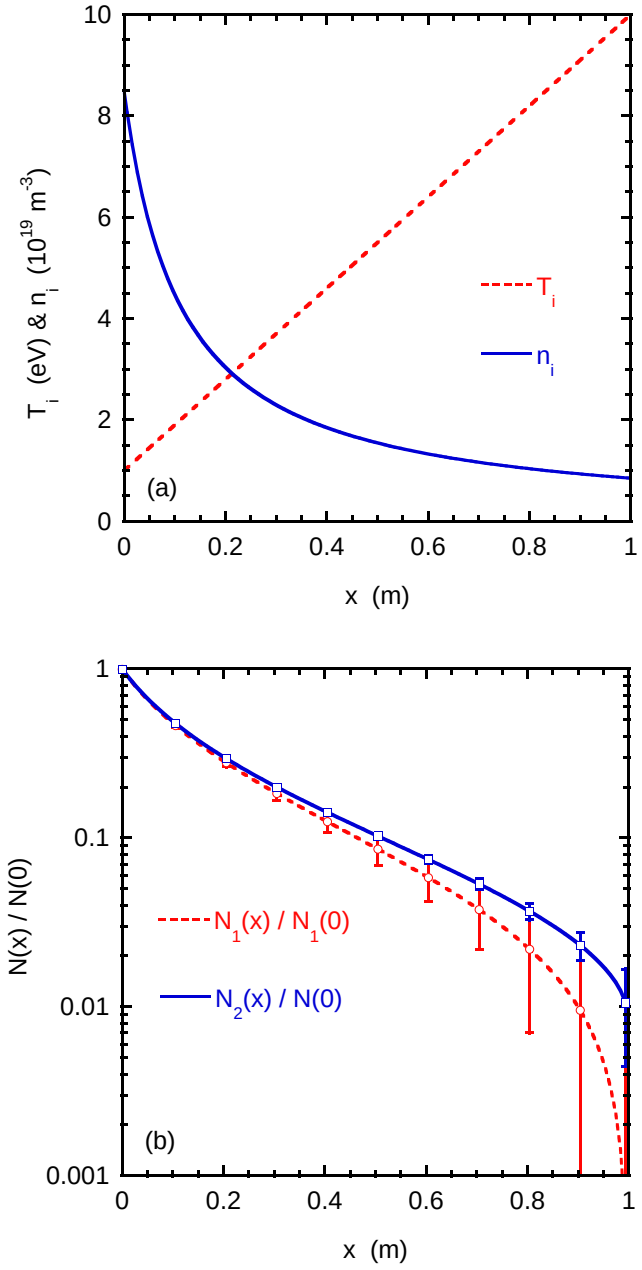

Figure 1: (a) Assumed ion density and temperature profiles and (b) normalized neutral density variation with x in the two analytic solutions, Eqs. (4) and (7).